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Factor Substitution, Average Firm Size and Economic Growth

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ABSTRACT. This paper extends the Lucas (1978, *The Bell Journal of Economics* **9**(2), 508–523) analysis of firm size by taking into account a normalised aggregate CES production function. In a general equilibrium framework it is proved that there is an inverse relation between the elasticity of substitution and average firm size. If interpreted together with the fact that richer countries are characterised by a higher elasticity of substitution, this result can explain why the recent literature finds a positive association between the importance of SMEs in an economy and its stage of development, but seems to fail in finding causality between the two. Both have a common origin: a high value of the elasticity of substitution. This paper also provides a first empirical test of the theory proposed using cross-country data from both developed and developing countries.

KEY WORDS: average firm size, general equilibrium models, neoclassical growth models, CES function.

JEL CLASSIFICATION: C65, E13, L11.

1. Introduction

In a recent working paper Beck et al. (2003) present evidence that in a very large cross section of countries at different levels of economic development the importance of small and medium-sized enterprises (SMEs) is positively correlated with GDP per capita growth. However, they also point out that this relationship is not robust to controlling for simultaneity bias, while a large sector of SMEs is a peculiar characteristic of well-developed economies, there seems to be no evidence that SMEs are an engine of growth. Weeks (2003) qualifies this argument claiming that in developing countries SMEs' importance tends to decline at the very early stages of development but that this trend is reversed as income grows. He therefore points out a U-shaped pattern.

The result that developed countries are (now) characterised by a large sector of SMEs is not new in the literature. In the last four decades, the importance of SMEs across the globe has grown both in absolute and relative terms (Acs and Audretsch, 1993; Loveman and Sengerberger, 1991). This phenomenon is very surprising from a purely theoretical point of view since the conventional wisdom is that larger enterprises are more capable of exploiting scale economies and, therefore, as an economy develops they should gain incrementally in importance. SMEs were thought to be economically less efficient than large firms, less innovative and offering lower quality jobs.

The proposed explanations for these phenomena have been numerous: (1) technological change had reduced the extent of scale economies; (2) increased globalization had rendered markets more volatile and favoured small enterprises; (3) the changing composition of the labour force may be more conducive to small enterprises due to the higher premium placed on work flexibility; (4) changing consumer tastes facilitated some niche producers; (5) deregulation lowered entry-barriers; (6) increased importance of innovation in high wage countries fostered entrepreneurship and (7) the switch

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towards a knowledge-based economy promoted firm formation.[1]

In this paper we argue that a possible explanation of the increase in the importance of SMEs across the globe is related to the elasticity of (factor) substitution, which is already known as an important determinant of economic development. We use the well-known Lucas (1978) model of firm formation in order to analyse the effects of a changing elasticity of substitution on average firm size, and we prove that, under reasonable conditions, an increase in the (aggregate) elasticity of substitution tilts the size distribution of firms towards smaller firms. To this aim we use a normalised CES function in order to compare economies that differ only in relation to their factor substitutability. Since we regard the elasticity of substitution as a general measure of the flexibility of market systems (Klump, 2001), its increase can reflect external liberalization as well as internal deregulation.

The paper proceeds as follows: Section 2 discusses determinants of factor substitution and empirical evidence of the aggregate substitution elasticity. Section 3 presents our basic model, which builds on Lucas (1978) and introduces the normalized CES production function. Section 4 analyzes the influence of factor elasticity on average firm size and the level of development. Section 5 contains a preliminary test of our theory in which we take a measure of openness to international trade as a proxy for factor substitution. Section 6 summarizes and concludes.

2. The elasticity of factor substitution: determinants and empirical evidence

Factor substitution between capital and labour is an important determinant of economic development which has been neglected for a long time in neoclassical growth theory (Klump and De La Grandville, 2000; Klump and Preissler, 2000). Hicks (1963) has pointed out three different ways in which the substitution between factors of production takes place: (a) inter-sectoral substitution of production, (b) intra-sectoral substitution of the known methods of production and (c) substitution by innovation. These three ways determine how flexible a market economy with given input factors can react to external shocks. The elasticity of substitution measures quantitativley the degree of factor substitution and the flexibility of a country's productive system which represents "the 'menu of choice' available to entrepreneurs" (Yuhn, 1991, p. 344).

Determinants of the elasticity of substitution can be found in various external and internal factors governing the flexibility of the market system (Klump and Preissler, 2000, 51 ff.). Following Ventura (1997) and building on ideas which were already formulated in classical economics, an obvious determinant of the aggregate elasticity of substitution is the degree of trade liberalization. By opening up to foreign trade and managing the induced structural change, a country is able to overcome the growth restrictions imposed by the scarcity of single input factors. It can then be modelled as possessing a (de facto) higher degree of factor substitution.

Other determinants relate to the internal regulation of factor reallocation and the institutional support to innovative activities. Maki and Meredith (1987) concluded from an investigation of Canadian industries that a high degree of unionization leads to low elasticities of substitution, both at the sectoral and the aggregate level. Cronin et al. (1997) estimated sectoral elasticities of substitution across US industries. They found high values in many service sectors where the influence of trade unions is lowest and in those sectors where public regulations preventing intra- and inter-sectoral reallocations are absent. Weder and Grubel (1993) point out that the existence of private institutions fostering innovation, such as research associations, clusters of innovative industries, strategic alliances of firms and so on, can cause high elasticities of substitution. This would happen both because more innovations would be produced in this framework and because more inter- and intra-sectoral substitution would take place.

Klump et al. (2004) provide an overview of the empirical evidence on aggregate elasticities of substitution in developed and developing countries. For the US the earliest econometric estimation of the (aggregate) elasticity of substitution (Arrow et al. 1961) had obtained a value well below unity. This result had been challenged by Berndt (1976), who found that at least in the US manufacturing sector, the elasticity of substitution seemed to be roughly equal to one. The most recent studies are more supportive of the earlier view. Chirinko (2002) summarising a number of different estimates based on US industry data concludes that the elasticity of substitution is significantly lower than one. Antras (2004) shows that those estimations for the US, which accepted the unitary value, suffered from an omitted variable bias. Also for other industrial countries the elasticity of factor substitution is generally estimated at a value below one. In a cross-study study covering developing and developed countries, Duffy and Papageorgiou (2000) as well as Pereira (2002) find evidence that a higher level of development goes hand in hand with a higher elasticity of substitution.

3. Entrepreneurial talent, factor substitution and average firm size: the Lucas model with a normalized CES technology

In order to explain average firm size in a general equilibrium context we use the model of Lucas (1978).[2] This model has two particular features: a given production technology and a given distribution across individuals of entrepreneurial talent.[3] Every individual can either be employed in an existing firm or will start a new firm. If she chooses to start a new firm she can produce an output using n units of labour and k units of capital via a production technology with constant returns to scale, $f(n,k) = n\phi(r)$, where $r = k/n$. We specify this production technology as a normalized CES production so that the influence of factor substitution on firm size can be studied explicitly.

The normalised CES function was introduced by Klump and De la Grandville (2000) and Klump and Preissler (2000) and can be written as[4]:

$$\phi = A(\sigma)\left\{a(\sigma)r^{\psi} + (1 - a(\sigma))\right\}^{1/\psi} \tag{1}$$

where $\psi = (\sigma - 1)/\sigma$; $a(\sigma) = r^{\circ 1-\psi}/(r^{\circ 1-\psi} + m^{\circ})$ and $A(\sigma) = y^{\circ}\left((r^{\circ 1-\psi} + m^{\circ})/(r^{\circ} + m^{\circ})\right)^{1/\psi}$ are the substitution, distribution and the efficiency parameter, respectively. In contrast to other CES specifications this particular functional form allows to compare economies that only differ in respect to their factor substitutability. The normalised CES function is defined by baseline values for the per-capita capital stock r°, per capita production $y^{\circ} = \phi(r^{\circ})$ and the marginal rate of substitution $m^{\circ} = [\phi(r^{\circ}) - r^{\circ}\phi'(r^{\circ})]/\phi'(r^{\circ})$ in a point that is common to all functions with different elasticities of substitution. This normalization point can be regarded as the origin of a system of functions, which are only distinguished by different elasticities of substitution. The baseline values can be chosen arbitrarily. Sometimes the baseline values for r and y are for simplicity set equal to one[5] which reflects the idea that the values determining the origin of a system of functions should be rather low. By normalization the efficiency and distribution parameters of the CES function, A and a, are no longer considered as given, but become dependent on the elasticity of substitution and the baseline values.

Output depends in the Lucas model also on a parameter x which represents differences in managerial talent across individuals and a function g. The parameter x is drawn from a fixed distribution $\Gamma : R^{+} \to [0,1]$ of managerial talent. Those individuals who lie at the top end of the spectrum, i.e. the most talented managers, run firms and become entrepreneurs; the others prefer to be employees and work for them. The distribution of talents affects output so that if agent x manages n units of labour and k units of capital her firm produces $xg[f(k,n)]$ units of output where $g(k,n)$ is an increasing, strictly concave function satisfying $g(0) = 0$. The concavity assumption is needed to avoid the trivial solution that the individual who lies at the top of the spectrum (the best manager) manages all other individuals. Following Lucas (1978), we assume that Gibrat's law holds, i.e., that firm size and firm growth are independent. Then the function g can be specified as[6]:

$$g(k,n) = \alpha[n\phi(r)]^{\beta} \tag{2}$$

where α and β are two constants.

Assuming that the entire distribution of x is always fully represented, a resource allocation is described by two functions $n(x)$ and $k(x)$ that describe the amount of labour and capital managed by agent x. An allocation will require that for some agents who are labourers $n(x) = k(x) = 0$, while $n(x) > 0, k(x) > 0$ for those agents who become entrepreneurs.

It is therefore possible to obtain a cut-off point z such that if $x > z$ one individual is an entrepreneur and if $x < z$ another individual is an employee.

An efficient allocation is one which maximises output per capita:

$$\frac{Y}{N} = \int_z^\infty xg[f(n(x), k(x))]d\Gamma(x) \tag{3}$$

subject to the two constraints (4) and (5):

$$1 - \Gamma(z) + \int_z^\infty n(x)d\Gamma(x) \leq 1 \tag{4}$$

so that the share of people who are entrepreneurs plus the share of people who are employees is smaller or equal to one and

$$\int_z^\infty k(x)d\Gamma(x) \leq \frac{K}{N} = R \tag{5}$$

so that no more than the entire amount of capital is employed in the production process.

Together with the assumption (2) reflecting Gibrat's law one obtains the following expression for per capita output:

$$\frac{Y}{N} = \alpha(\phi(r))^\beta(\Gamma(z))^\beta(L(z))^{1-\beta} \tag{6}$$

where $L(z) = \int_z^\infty x^{1/(1-\beta)}d\Gamma(x)$

Maximization of efficient output (6) with respect to z leads to the following first-order condition:

$$\frac{L(z)}{\Gamma(z)}\beta[1 - \pi(r)] = (1 - \beta)z^{1/(1-\beta)} \tag{7}$$

where

$$\pi(r) = r\frac{\phi'(r)}{\phi(r)} = \frac{r^\psi r^{\circ 1-\psi}}{r^\psi r^{\circ 1-\psi} + m^\circ} \tag{8}$$

is the capital income share calculated from the normalized CES production function.

Both sides of (7) depend on z. The expression on the right-hand side of (7) passes through the origin and is increasing (and convex). On the left-hand side L/Γ tends to zero as z tends to infinity and vice versa. Assuming that $1 - \pi(r)$ remains bounded away from 0 and 1 the asymptotic behaviour of the left-hand side of (7) is that of L/Γ. Both parts of (7) are drawn in Figure 1, where z^* represents the equilibrium cut-off point, that is the managerial talent of the marginal manager. Figure 1 can therefore be used to analyse average firm size in an economy. If z^* is large then the average firm is large as well, since there will be more employees and fewer managers: only individuals with a managerial talent sufficiently large will run firms. In contrast, if z^* is low then average firm size will

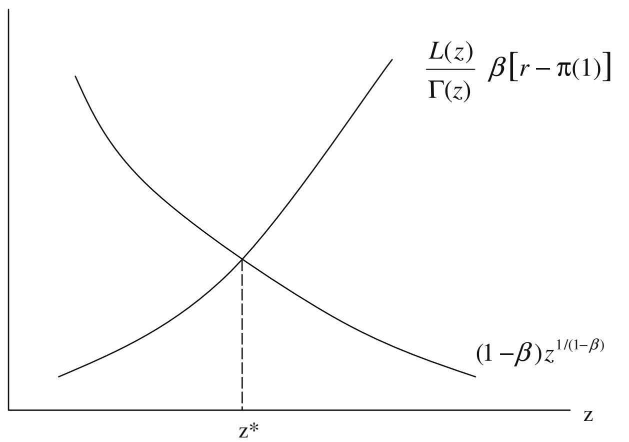

Figure 1. The equilibrium cut-off point.

be small since there is little managerial talent required to run a firm.

4. Factor substitution, capital accumulation and firm size

Since the elasticity of substitution only appears in the capital income share (8) it is via a change in π that factor substitution influences average firm size. Two possible cases can be distinguished: a first one induced by a change in r and a second one caused by a change in σ. Lucas (1978) had already studied the first case without further specifying the CES production technology. He conjectured that for $\sigma < 1$ an exogenous increase in per capita capital shifts the falling curve in Figure 2 (representing the left-hand side of Equation (7)) upwards and thus leads to a larger equilibrium firm size represented by z_1^*. We can prove this conjecture by calculating from (8) that[7]:

$$\frac{\partial\pi}{\partial r} = \psi\frac{\pi}{r}(1-\pi) < 0, \quad \text{if } \sigma < 1 \tag{9}$$

This result, that richer economies should be characterized by a larger average firm size, relies on the reaction of factor income shares and factor prices. An increase in per capita capital raises the wedge between the wage rate and the return to managerial activities, pushing individuals out of entrepreneurship and into paid work and thereby increasing average firm size.

The second case, which studies the direct influence of the elasticity of substitution on the capital income share, can also be derived from (8). As shown in Klump and De La Grandville (2000, p. 285) one obtains:

$$\frac{\partial\pi}{\partial\sigma} = \frac{1}{\sigma^2}(1-\pi)\pi\ln\left(\frac{r}{r^\circ}\right) \tag{10}$$

If we regard r° as the baseline value at the origin of our system of normalized CES functions we can concentrate on cases where $r > r^\circ$ and hence $\partial\pi/\partial\sigma > 0$. This means that in Figure 2 an increase in the elasticity of substitution shifts the falling curve to the left and therefore reduces average firm size to z_2^*. The logic behind this result can be summarised as follows. Recalling that in the model only the "best" managers actually start their own firm, the easier it is to substitute factors the easier it is to be an entrepreneur, i.e., the managerial talent required to run a firm is lower. An individual with a relatively low endowment of entrepreneurial talent can start an own firm only if it is relatively easy to substitute the factors of production.

The two cases of comparative statics in which the elasticity of substitution is involved seem to lead therefore to contrary results concerning average firm size. It should be noted, however, that the second case has so far been analyzed only in a partial equilibrium because per capita capital has been treated as given. As shown in Klump and De La Grandville (2000) an increase in the elasticity of substitution does also lead to higher per capita capital in the steady state of a standard neoclassical model. Following the result of the first case – higher

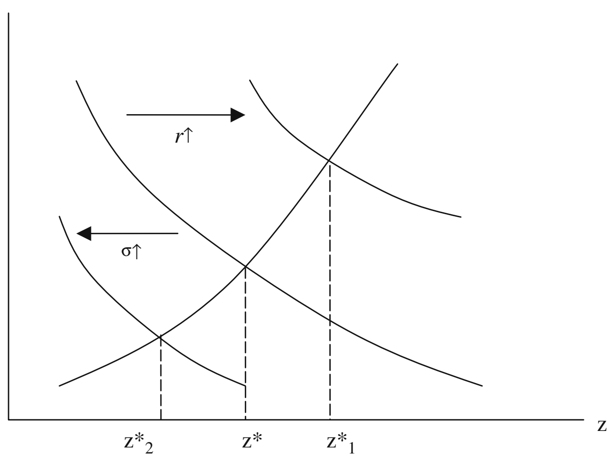

Figure 2. Comparative statics.

per capita capital increases average firm size – this second-order effect works against the negative relationship between factor substitution and average firm size. We have to find out which effect dominates.

Since the Lucas model is a variant the neoclassical Solow (1956) growth model we can determine the steady state by introducing a constant savings rate s and the constant growth rate of the labour force $\eta = \dot{n}/n$. As long as saving is independent from income distribution the size distribution of firms does not influence capital accumulation but depends on it. We can characterize the steady state of the complete model by the steady state capital income share, which has the following form[8]:

$$\pi_{ss} = \left(\frac{s}{\eta}\frac{y^\circ}{r^\circ}\right)^{\psi} \pi^\circ \tag{11}$$

The derivative of the steady state capital income share with respect to σ can be calculated as:

$$\frac{\partial \pi_{ss}}{\partial \sigma} = \frac{1}{\psi}\frac{1}{\sigma^2}\pi_{ss}\ln\left(\frac{\pi_{ss}}{\pi^\circ}\right) = \frac{1}{\sigma^2}\pi_{ss}\ln\left(\frac{s}{\eta}\frac{y^\circ}{r^\circ}\right) \tag{12}$$

This expression is clearly positive, which would underline the negative relationship between elasticity of substitution and average firm size, if the condition

$$sy^\circ > \eta r^\circ \tag{13}$$

holds. This condition is, however, satisfied in the standard neoclassical growth model as long as the baseline values are set at such low levels that the steady state is obtained at higher values of development. Figure 3 derived from the well-known Solow growth model illustrates this point. If we regard the baseline values r° and y° as determining the origin of the system of normalized CES functions the steady state will always be located to the right. In the origin the system will then experience positive growth towards the steady state.

Let us briefly summarize the theoretical findings: an increase of the elasticity of substitution, which can be caused by deregulation or by integration into international markets, will lead in our model at the same time to higher per-capita capital and lower average firm size. Therefore, having a well-developed SMEs sector and being a good performer in terms of growth rates can have a common origin, namely high factor substitutability. This very simple consideration can help us understand the results of Beck et al. (2003), who state that although there is evidence that more advanced countries are characterised by a more important SMEs sector there is no causal relationship between SMEs and growth. In Section 5 we provide a preliminary test of our model by regressing both growth rates and two indicators of SMEs' importance on a set of control variables that our

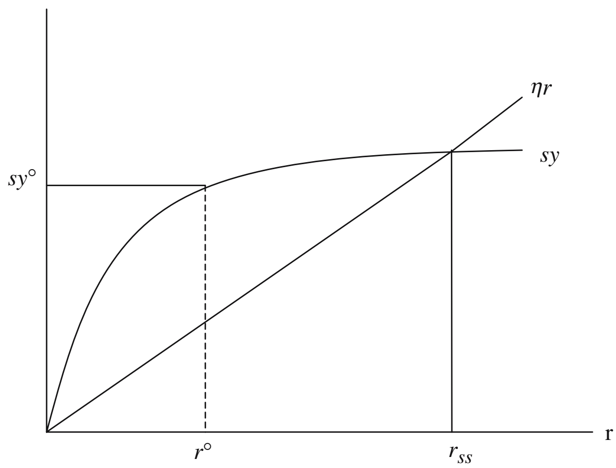

Figure 3. Baseline values and steady state.

model predicts to be crucial in explaining these phenomena.

5. A preliminary test

Testing the implications of the model proposed in Sections 3 and 4 is by no means an easy task. An appropriate test would require data on the birth and death of firms and the evolving dynamics of average firm size to be related to some indicators of the elasticity of substitution and to a set of control variables. However, such a strategy is not feasible for a number of reasons: first of all it is difficult to find a panel of firms which is including new firms and eliminating the firms that die; second, it is very difficult to find reliable estimates of the elasticity of substitution. These estimates seem to be heavily dependent on the assumption of a particular form taken by technological change,[9] and furthermore they are very often limited to the manufacturing sector. Solving some of these problems will require further research. Thus, in this study we have been forced to adopt an alternative strategy, namely a cross-country analysis, which has to be regarded as a first test and it is therefore not conclusive. We focus only on the steady state implication of our model since the data we use do not have a dynamic dimension.

The data of SMEs' importance come from the Ayyagari et al. (2003) working paper. They provide two measures of SMEs' importance for the 1990s that are SME_OFF and SME_250, which represent the share of formal employment in the SMEs sector according to the official definition of each country and to a common value of 250 employees (where available) respectively. We use the Sachs and Warner (1995) indicator for international openness[10] as a measure of the (aggregate) elasticity of substitution in the spirit of Ventura's (1997) approach mentioned above. This measure is a dummy variable which takes the value of one if four conditions hold: (1) average tariff rates below 40%; (2) average quota and licensing coverage of imports of less than 40 percent; (3) a black market exchange rate premium that averaged less than 20 percent during the decade of the 1970s and 1980s; and (4) no extreme controls (taxes, quotas, state monopolies) on exports. We average these variables through the 1980s to obtain an indicator of openness (i.e., value of σ), which varies between zero and one (SW80). As control variables we use the initial per capita GDP (IGDP90) to proxy for the initial development level (from the World Development Indicators; World Bank, 2004), the fertility rate (FERT) to proxy for population growth[11] and investment as a share of GDP (from the Penn World Table Version 6.1, Heston et al., 2002) to proxy for savings and capital formation. As for FERT and SW80 we used lagged values while for investment an IV estimator (IVINV) has been used[12] to avoid possible endogeneity problems. All the variables are decade averages from either the 1980s or the 1990s. The number of observations available for each regression varies between a maximum of 72 and a minimum of 37.

We perform a growth regression[13] and a regression on SMEs' importance. The models used can therefore be summarised in the following equation:

$$X = \alpha_0 + \alpha_1 \mathrm{IGDP90} + \alpha_2 \mathrm{FERT} + \alpha_3 \mathrm{IVINV} + \alpha_4 \mathrm{SW80} + \varepsilon$$

where X represents either growth rates in the 1990s (GROWTH90) from the Penn World Table (Heston et al., 2002) or one of the two indicators of SMEs' importance. Table I summarizes the results.

The regressions appear to confirm preliminarily the theory presented in our paper: the openness indicator is positively correlated with both growth rates and SMEs importance. As for the fertility rate and investment the sign of the two coefficients is the predicted one: negative for the first one and positive for the second one, although only the second is significant at 5% level.

It looks like the "common origin" between higher growth rates and SMEs' importance is confirmed by this basic test.

6. Summary and conclusions

In this paper we analyse the effects of a changing elasticity of substitution on average firm size in a general equilibrium model. We prove that a higher (lower) value of the

TABLE I
An empirical test of the effects of the elasticity of substitution on growth and SMEs importance

Dep. variable	GROWTH90	SME_OFF	SME_250
Sample mean of dep. variable	1.65	51.44	53.28
No. of observations	72	50	37
R^2	0.2809	0.5323	0.6259
F. stat	6.54	12.80	13.39
Prob > F	0.0002	0.0000	0.0000
Constant	−0.95918	45.6601***	46.29721***
	(−0.54)	(3.00)	(2.89)
IGDP90	−9.45e−05***	−0.0006905**	−0.0010469***
	(−2.98)	(−2.68)	(−3.23)
FERT	−0.0064	−3.085173	−3.929388*
	(−0.03)	(−1.56)	(−1.85)
IVINV	0.15072**	1.352505**	1.507698**
	(2.12)	(2.27)	(2.38)
SW80	1.79207**	12.32345**	18.01331**
	(2.58)	(2.06)	(2.52)

Note: *t*-values are in parentheses, *** significant at 1% level,** significant at 5% level, * significant at 10% level. All standard tests have been performed, models have homoskedastic and normally distributed disturbances.

elasticity of substitution favours a lower (higher) average firm size. We also explain the puzzling empirical evidence that SMEs and income growth rates are positively correlated, but without a clear causal link. Our basic framework is that of the well-known Lucas (1978) model of firm formation in which individuals differ in respect to their managerial ability. We extend the Lucas analysis by introducing a normalized CES production technology, which allows us to treat the elasticity of substitution as an explicit parameter of the model. We consider the elasticity of substitution as a measure of efficiency of the market system which is influenced by, e.g., internal regulations and external trade barriers. The same factors have also been emphasized as relevant determinants of SMEs importance in the literature, but so far without a solid theoretical elaboration.

Lucas has derived from this model the conclusion that higher development leads to higher average firm size. We come to a different conclusion. A high value of σ does not only lead to more per capita capital, but makes it at the same time easier for an individual to become an entrepreneur. Therefore, in an economy characterised by higher values of σ we should expect a higher level of development, more entrepreneurs and smaller firms. We present a first and very preliminary empirical test of our hypothesis which finds evidence of a negative relationship between a measure of openness, considered as a proxy for the elasticity of substitution, and average firm size in a cross-section of (around) fifty developed and developing countries.

Two extensions of our approach seem to be promising. Our results could be compared to results from other models where individuals differ in respect to a parameter which affects the marginal product of labour and total output in the same way (e.g., Kihlstrom and Laffont, 1979, for a model with risk aversion). The steady state results of our model rely on the critical assumption that saving and investment is not influenced by the distribution of income. With different saving rates for different income shares the model might generate interesting multiple equilibria which could be helpful explaining the U-shaped patterns of SME importance in development described by Weeks (2003).

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Amsterdam and an anonymous referee for helpful suggestions. Financial support from Istituto Veritatis Splendor of the Fondazione Lercaro is gratefully acknowledged. Needless to say, the usual disclaimer applies.

Appendix

A.1. The derivation of the Lucas model

In this appendix we provide a derivation of the Lucas (1978) model in order to obtain Equation (6). Starting from Equation (3) in the text and assuming that the Lagrangian multipliers associated with the constraints (4) and (5) are the wage rate w and the returns to capital u an efficient allocation will also be a competitive equilibrium. The first-order conditions of this maximisation problem are:

$$xg'(f)f_n(n(x),k(x)) = w, \quad x \geq z \tag{A.1}$$

$$xg'(f)f_k(n(x),k(x)) = u, \quad x \geq z \tag{A.2}$$

That taken together, implies:

$$\frac{w}{u} = \frac{\phi(r) - r\phi'(r)}{\phi'(r)} \quad \text{since} f(n,k) = n\phi(r) \tag{A.3}$$

Given r from (A.3) it is possible to obtain the equilibrium size $n(x)$ of firm x from either (A.1) or (A.2). Using (A.2) we get:

$$xg'[n(x)\phi(r)]\phi'(r) = u \tag{A.4}$$

Equation (A.4) gives firm size, measured by employment, as an implicit function $n(x,w,u)$ if $x \geq z$. The function $n(\cdot)$ is increasing in x and u and decreasing in w.

The two conditions (A.1) and (A.2) can be easily rewritten for the "marginal" manager by substituting z in the equations and calculating in a straightforward manner a unique employment level for the marginal manager $n(z,w,u)$.

With this model it is fairly easy to calculate an equilibrium solution if both capital per capita and the distribution of x are fixed. However, to analyse the dynamics of the model, since the problem is not concave Lucas, assumes the validity of Gibrat's Law and proves that g has the form given by Equation (2). Using this equation and (A.4) we get:

$$n(x,w,u) = \frac{1}{\phi(r)}\left[\frac{u}{\alpha\beta x\phi'(r)}\right]^{1/(\beta-1)} \tag{A.5}$$

Then bearing in mind that $k(x,w,u) = rn(x,w,u)$, where r is given by (A.3), inserting $n(x,w,u)$ (from (A.5)) and $k(x,w,u)$ into the constraints (4) and (5) yields:

$$\left[\frac{u}{\alpha\beta\phi'(r)}\right]^{1/(1-\beta)}\frac{1}{\phi(r)}L(z) = \Gamma(z) \tag{A.6}$$

and

$$r\Gamma(z) = R \tag{A.7}$$

$$\text{where } L(z) = \int_z^\infty x^{1/(1-\beta)}d\Gamma(x) \tag{A.8}$$

It is now possible to express per capita output Y/N as a function of z and R, by inserting the employment solution (A.5) into (3) we get: $\frac{Y}{N} = \alpha\left[\frac{\alpha\beta\phi'(r)}{u}\right]^{1/(1-\beta)}L(z)$, now using (A.6) and (A.7) we can write:

$$\frac{Y}{N} = \alpha(\phi(r))^{\beta}(\Gamma(z))^{\beta}(L(z))^{1-\beta} \tag{A.9}$$

which is Equation (6) in the text.

A.2. The normalised CES production function

Starting from a standard CES function $\phi = A(ar^{\psi} + (1-a))^{1/\psi}$ where $\psi = (\sigma - 1/\sigma)$ let us take three, arbitrarily chosen, baseline values for per capita capital r°, per capita production $y^{\circ} = \phi(r^{\circ})$ and the marginal rate of substitution $m^{\circ} = [\phi(r^{\circ}) - r^{\circ}\phi'(r^{\circ})]/\phi'(r^{\circ})$

We obtain a system of equations in A and a:

$$m^{\circ} = \frac{1-a}{a}r^{\circ 1-\psi}$$

$$y^{\circ} = A[ar^{\circ\psi} + (1-a)]^{1/\psi}$$

and for given values of r°, y° and m° we can obtain the two parameters A and a as a function of σ:

$$a = \frac{r^{\circ 1-\psi}}{r^{\circ 1-\psi} + m^{\circ}} = a(\sigma; r^{\circ}, m^{\circ}) \equiv a(\sigma)$$

$$A = y^{\circ}\left(\frac{r^{\circ 1-\psi} + m^{\circ}}{r^{\circ} + m^{\circ}}\right)^{1/\psi} = A(\sigma; r^{\circ}, m^{\circ}, y^{\circ}) \equiv A(\sigma)$$

We obtain the normalised CES function as:

$$\phi = A(\sigma)\{a(\sigma)r^{\psi} + (1 - a(\sigma))\}^{1/\psi} \qquad \text{(A.10)}$$

The most popular forms of the normalised CES functions work with a normalisation in which either $r^{\circ} = \left(\frac{k}{n}\right)^{\circ} = 1$ or even $k^{\circ} = n^{\circ} = 1$. These normalisations are also implicitly contained in every standard Cobb–Douglas production function (see Klump and Preissler, 2000, p. 46, Table 2).

A.3. The profit share and its derivatives with respect to r and σ

Calculating the profit share using the normalised CES yields:

$$\pi = \frac{r\phi'_{\sigma}(r)}{\phi_{\sigma}(r)} = \frac{ar^{\psi}}{ar^{\psi} + (1 - a)} = a\left[\frac{Ar}{\phi_{\sigma}(r)}\right]^{\psi}$$
$$= \frac{r^{\psi}}{r^{\psi} + (1 - a)/a} = \frac{r^{\psi}r^{\circ 1-\psi}}{r^{\psi}r^{\circ 1-\psi} + m^{\circ}} \qquad \text{(A.11)}$$

while the baseline profit share (for $r = r^{\circ}$) can be written:

$$\pi^{\circ} = a\left[\frac{Ar^{\circ}}{y^{\circ}}\right]^{\psi} = \frac{r^{\circ}}{r^{\circ} + m^{\circ}} \qquad \text{(A.12)}$$

The derivative of the profit share with respect to r has therefore the form:

$$\frac{\partial\pi}{\partial r} = \psi\frac{r^{2\psi-1}r^{\circ 2(1-\psi)} + m^{\circ}r^{\psi-1}r^{\circ 1-\psi} - r^{2\psi-1}r^{\circ 2(1-\psi)}}{(r^{\psi}r^{\circ 1-\psi} + m^{\circ})^2}$$
$$= \psi\frac{1}{r}\frac{r^{\psi}r^{\circ 1-\psi}m^{\circ}}{(r^{\psi}r^{\circ 1-\psi} + m^{\circ})^2} = \psi\frac{\pi}{r}(1 - \pi) \qquad \text{(A.13)}$$

which is Equation (9) in the text.

On the other hand the derivative of the profit share with respect to sigma has the form:

$$\frac{\partial\pi}{\partial\sigma} = \frac{1}{\sigma^2}\frac{\partial\pi}{\partial\psi} = \frac{1}{\sigma^2}\frac{1}{(r^{\psi}r^{\circ 1-\psi} + m^{\circ})^2}$$
$$\times [r^{\psi}r^{\circ 1-\psi}(r^{\psi}r^{\circ 1-\psi} + m^{\circ})$$
$$(\ln r - \ln r^{\circ}) - r^{\psi}r^{\circ 1-\psi}r^{\psi}r^{\circ 1-\psi}(\ln r - \ln r^{\circ})]$$
$$= \frac{1}{\sigma^2}\frac{r^{\psi}r^{\circ 1-\psi}m^{\circ}}{(r^{\psi}r^{\circ 1-\psi} + m^{\circ})^2}\ln\left(\frac{r}{r^{\circ}}\right)$$
$$= \frac{1}{\sigma^2}(1 - \pi)\pi\ln\left(\frac{r}{r^{\circ}}\right) \qquad \text{(14)}$$

which is Equation (10) in the text.

A.4. The steady state profit share and its derivative with respect to σ

Since in the Solow model the steady state per capita capital can be written as:

$$r_{ss} = \left[\frac{1 - a}{\left(\frac{\eta}{sA}\right)^{\psi} - a}\right]^{1/\psi} \qquad \text{(A.15)}$$

Then we can compute the steady state profit share using (A.12) as:

$$\pi_{ss} = a\left[\frac{Ar_{ss}}{\phi_{\sigma}(r_{ss})}\right]^{\psi} = a\left(\frac{sA}{\eta}\right)^{\psi} = \frac{r^{\circ 1-\psi}}{r^{\circ} + m^{\circ}}\left(\frac{sy^{\circ}}{\eta}\right)^{\psi}$$
$$= \pi^{\circ}\left(\frac{sy^{\circ}}{\eta}\right) \qquad \text{(A.16)}$$

From (A.16) we can compute the derivative of the steady state profit share with respect to σ:

$$\frac{\partial\pi_{ss}}{\partial\sigma} = \frac{1}{\sigma^2}\pi_{ss}\left[\ln\left(\frac{sy^{\circ}}{\eta}\right) - \ln r^{\circ}\right] = \frac{1}{\sigma^2}\pi_{ss}\ln\left(\frac{sy^{\circ}}{\eta r^{\circ}}\right) \qquad \text{(A.17)}$$

which is Equation (12) in the text.

Notes

[1] See Brock and Evans (1989), Audretsch and Thurik (2001), Audretsch (2002) and Pietrobelli et al. (2004) *inter alia.*

[2] A complete derivation of the model can be found in Appendix A.1.

[3] Kihlstrom and Laffont (1979) construct a theory of firm formation which is very similar to the Lucas model. Individuals are not characterised by differences in managerial talent but they only differ in respect to their risk aversion. They have to choose between starting a new firm and

getting an uncertain return to their entrepreneurial activity or join an existing firm and get a certain wage. In equilibrium more risk averse individuals will be employees of firms run by the less risk averse ones.

[4] See Appendix A.2 for the derivation of a normalised CES function.

[5] See, e.g., Rowthorn (1999) and Acemoglu (2002) where $y^\circ = r^\circ = 1$ is implicitly assumed or Antras (2004) who works with the implicit assumption that $r^\circ = 1$.

[6] See Lucas (1978 pp. 514–515) for a detailed derivation of this result.

[7] See Appendix A.3 for the detailed calculations.

[8] This result has been derived by Klump and De la Grandville (2000). Appendix A.4 reports the proof for completeness.

[9] See Table I in Klump et al. (2004)

[10] The choice of the measure is clearly an arbitrary one, however with a cross section of countries where none of them can influence international markets alone the degree of openness can be regarded as a good approximation for the elasticity of substitution (Ventura, 1997).

[11] These data are World Bank staff estimates from various sources including the United Nations Statistics Division's Population and Vital Statistics Report, country statistical offices, and Demographic and Health Surveys from national sources and Macro International (SIMA).

[12] The variables in the first stage of the regression are: religious and legal origin variables; a democracy score; indicators of property rights and business environment (from Laporta et al., 1999), inflation rates and an indicator of schooling (from the World Development Indicators, World Bank, 2004), dummies for Latin America and Sub-Saharan Africa.

[13] Data on growth rates come from the Penn World Table Version 6.1 (Heston et al., 2002).

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